Optimal Positioning Strategies for Shape Changes in Robot Teams

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***Abstract*— In this paper, we consider the task of repositioning a formation of robots to a new shape while minimizing either the maximum distance that any robot travels, or the total distance traveled by the formation. We show that optimal solutions in $SE(2)$ can be achieved for either metric through second-order cone programming (SOCP) techniques. For the case where the orientation of the new formation shape is fixed, we obtain optimal solutions in both $\mathbb{R}^2$ and $\mathbb{R}^3$. The latter also allows for complete regulation of the formation size via constraints on the shape scale. We expect that these results will prove useful for extending the mission lives of robot formations and mobile ad-hoc networks (MANETs).**

***Index Terms*— Shape change, convex optimization, SOCP**

I. INTRODUCTION

Over the last several years, there has been a great deal of interest in developing *mobile ad-hoc networks* (MANETs) for applications such as environmental monitoring, health care, and homeland security to mention just a few. One of the major driving factors behind this interest is the new robotic applications originating from both the military and the civilian domains. In these roles, tasks may be extremely difficult for a single robot to accomplish. Thus, a system composed of teams of cooperative robots is desirable because of its flexibility, robustness and fault tolerance.

The research challenges encountered in multi-robot and sensor networks require the integration of different disciplines including biology, optimization, and control. Therefore, it is not surprising that the related literature enjoys the flavor of a broad spectrum of approaches which have been utilized for coordinating robot/sensor teams. Specifically, we are interested in developing optimization based positioning strategies for formation shape changes in robot teams. Changes in formation shape may be necessitated by new mission objectives, to compensate for node failures, or to accommodate for changes in the environment (*e.g.*, for obstacle avoidance). By formation *shape*, we are referring to the geometrical information that remains when location, scale, and rotational effects are removed. Thus, formation shape is invariant under the Euclidean similarity transformations of translation, rotation and scaling.

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The remainder of this paper is organized as follows. In section II, we review relevant work on formation control of robot teams and optimization approaches. Definitions and the problem formulation are given in section III. Sections IV and V give optimal solutions for the formation shape problem in $SE(2)$ and in $\mathbb{R}^3$ through SOCP, respectively. Additionally, these sections present simulation results that verify the validity of the proposed optimization strategies. Section VII reviews how motion constraints could be accommodated. The complexity of the proposed framework is discussed in section VIII. Finally, concluding remarks and future work are given in section IX.

II. RELATED WORK

Formations of robot teams have been extensively studied in the literature, and a complete survey is outside of the scope of this paper. Instead we focus on those where *shape* - defined differently under different contexts - was of significant relevance to the research.

Das *et al* described a vision-based formation control framework [1]. This focused on achieving and maintaining a given formation shape using a leader-follower framework. Control of formations using Jacobi shape coordinates was addressed by Zhang *et al* [2]. The approach was applied to a formation of a small number of robots which are modeled as point masses. Abstraction based control was used by Belta and Kumar as a mechanism to coordinate a large number of fully actuated mobile robots moving in formation [3]. The main idea was to map the configuration space of the robots $\mathcal{Q}$ to a lower dimensional manifold $\mathcal{A}$. The concept of *shape* refers to the area spanned by the robots. A local controller was designed based on the state of the robot and the state on the manifold $\mathcal{A}$.

Related studies include *conflict resolution* of multi-vehicle systems. In conflict resolution, trajectories are planned for vehicles operating independently in the same space but with potentially conflicting goals. The work of Ogren and Leonard used a dynamic window approach to avoid obstacles, and included the stability analysis [4]. These approaches often adopt simplified models for vehicle behavior, but present interesting applications for optimization-based methods.

In *cooperative control problems*, vehicles move in a coordinated fashion to achieve some common goal and/or seek to maintain some geometrical relationships among themselves. Often movement is dictated by measurement of gradients of some actual sensor measurements, or some artificial potential field. Solutions defined with inter-robot distance relationships were explored by Bachmayer and Leonard in [5], where methods to measure and project gradient information were discussed. The applications for these methods are in, for example, data acquisition in large areas such as oceans where the most advantageous arrangement of sensors may not be to distribute them evenly, but to have them adapt to concentrate more sensors in areas where the measured variable has steeper gradients.

There has also been significant interest in applying optimization based techniques to coordinate robot teams and deploy sensor networks. Contributions in this area include the work by Cortes *et al* [6]. Here the focus is on autonomous vehicles performing distributed sensing tasks. Recently Feddema *et al.* applied decentralized optimization based control to solve a variety of multi-robot problems [7]. Optimal motion planning was considered by Belta and Kumar [8]. In this work, authors generate a family of optimal smooth trajectories for a set of fully actuated mobile robots. The case for which robots have independent goals but share the same space has been studied by LaValle and Hutchinson in [9].

First, we should emphasize that while the primary focus of these efforts has been control, our results are more appropriate for use in a higher level planning phase. Given an initial formation configuration and a desired shape, we generate objective positions for each robot that will achieve the desired shape while minimizing the distances that the nodes must travel. These objective positions can then be fed to appropriate controllers to drive the nodes to their desired destinations. What further differentiates our work is that (with the exception of perhaps [8] which focused upon minimizing kinetic energy) the control policies for changing shape did not optimize over the distance traveled by each node. Lastly, our framework leverages recent advances in convex optimization and interior point methods (IPM) to solve the subsequent problems for large formations (> 1000 nodes) very quickly (*e.g.* ≈ 1 second) on current PC technology.

In this sense, our problem can be viewed as the converse to the assignment problem [10] where the final pose of the formation is known *a priori*, and the objective is to find an optimal assignment of nodes to objective positions. In contrast, we determine the optimal pose corresponding to the objective formation shape where assignments remain fixed across shape transitions.

III. Problem Formulation

The objective of this paper is to provide positioning strategies for robot teams to efficiently transition to a new formation *shape*, defined as follows:

Definition 3.1: The shape of a formation is the geometrical information that remains when location, scale, and rotational effects are removed.

Thus, formation shape is invariant under the Euclidean similarity transformations of translation, rotation and scaling [11].

Now consider a formation of $k \geq 2$ robots in a Euclidean Space $\mathbb{R}^m$, $m \in \{2,3\}$. Let $s_i \in \mathbb{R}^m$ denote the position of the i^{th} robot relative to some local frame $\mathcal{F}$. Without loss of generality, let s_1 correspond to the origin O. We denote the formation by the $k \times m$ matrix

$$S = [s_1, \ldots, s_k]^T \tag{1}$$

which is the concatenated coordinates of our k robots in $\mathcal{F}$. We can then define the shape of S as the equivalence class of the full set of similarity transformations of the formation

$$[S] = \{\alpha S R + 1_k d^T : \alpha \in \mathbb{R}_+, R \in SO(m), d \in \mathbb{R}^m\} \tag{2}$$

where $\alpha \in \mathbb{R}_+$ is the scale, $R \in SO(m)$ is a rotation matrix and $d \in \mathbb{R}^m$ is the translation vector. S then corresponds to an *icon* of the shape $[S]$.

Now let the $k \times m$ matrix $P = [p_1, \ldots, p_k]^T$ denote the concatenated coordinates of the current formation pose in some world frame $\mathcal{W}$. Our objective is to obtain a new formation pose $Q = [q_1, \ldots, q_k]^T$ in $\mathcal{W}$ where Q has the same shape as S under the equivalence relation defined in (2), and where either the maximum distance between the respective positions in P and Q are minimized, or where the sum of the distances is minimized. We can now formulate our problem:

Problem 3.2: Given an initial formation pose P in $\mathcal{W}$ and a formation shape icon S in $\mathcal{F}$, $P \not\sim S$, obtain a new formation pose Q in $\mathcal{W}$, $Q \sim S$, and where:

1) $\max \parallel q_i - p_i \parallel$ is minimized for $i = 1, \ldots, k$ OR
2) $\sum_{i=1}^{k} \parallel q_i - p_i \parallel$ is minimized

IV. Optimal Solutions in $SE(2)$ through Second-order Cone Programming (SOCP)

We first consider the case for operations limited to $SE(2)$ (on the plane). We can then represent the shape of our icon S as

$$[S] = \{\alpha S R + 1_k d^T : \alpha \in \mathbb{R}_+, R \in SO(2), d \in \mathbb{R}^2\} \tag{3}$$

Let $s_i = (s_i^x, s_i^y)$ and $q_i = (q_i^x, q_i^y)$ denote Cartesian coordinates in $\mathcal{F}$ and $\mathcal{W}$, respectively. Equation 3 then represents a set of equality constraints of the form

$$q_i^x - q_1^x = \alpha\,(s_i^x \cos\theta - s_i^y \sin\theta) \tag{4}$$

$$q_i^y - q_1^y = \alpha\,(s_i^x \sin\theta + s_i^y \cos\theta) \tag{5}$$

for $i = 2, \ldots, k$ and where θ corresponds to the orientation of the formation. Without loss of generality, we can define the formation orientation as

$$\theta = \arctan \frac{q_2^y - q_1^y}{q_2^x - q_1^x} \tag{6}$$

from which we obtain

$$\cos\theta = \frac{q_2^x - q_1^x}{\parallel q_2 - q_1 \parallel} \tag{7}$$

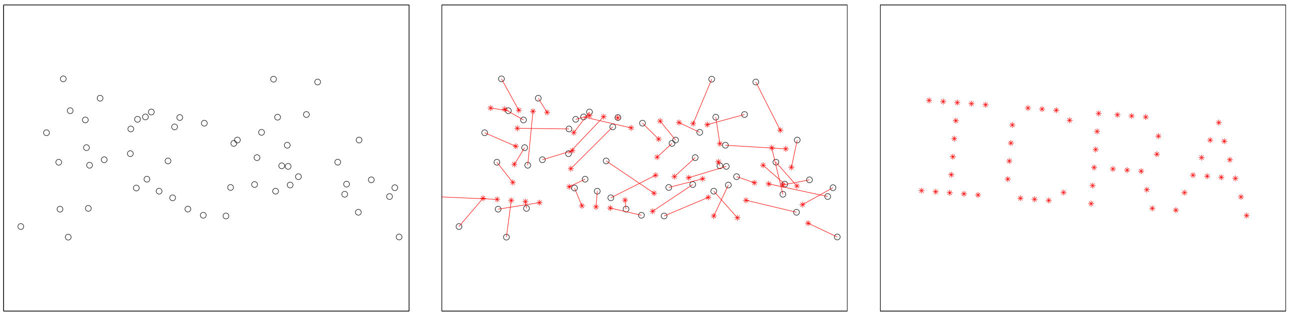

Fig. 1. The initial formation pose for a set of 55 nodes in $\mathbb{R}^2$ (left). The final formation trajectory/pose that achieves the desired shape while minimizing the maximum distance that any node must travel (center, right). The optimal values were 0.93 for scale, -4.6 degrees for orientation and $(0.41, 1.10)$ for translation.

$$\sin\theta = \frac{q_2^y - q_1^y}{\parallel q_2 - q_1 \parallel} \tag{8}$$

Noting that for $\alpha \geq 0$ and $s_1 \triangleq O$,

$$\alpha = \frac{\parallel q_2 - q_1 \parallel}{\parallel s_2 \parallel} \tag{9}$$

we can rewrite the constraints in (4)-(5) as

$$q_i^x - q_1^x = \frac{s_i^x}{\parallel s_2 \parallel}(q_2^x - q_1^x) - \frac{s_i^y}{\parallel s_2 \parallel}(q_2^y - q_1^y) \tag{10}$$

$$q_i^y - q_1^y = \frac{s_i^x}{\parallel s_2 \parallel}(q_2^y - q_1^y) + \frac{s_i^y}{\parallel s_2 \parallel}(q_2^x - q_1^x) \tag{11}$$

for $i = 3, \ldots, k$.

These $2(k-2)$ constraints are now convex (in fact, linear) functions of our state vector $q = (q_1, \ldots, q_k)^T$. They are also *necessary and sufficient* for describing the formation shape [12]. The four free variables correspond to the translation, rotation, and scale as defined in (3).

The problem of finding the formation pose Q can now be posed as the constrained optimization problems

$$\begin{array}{ll} \min\limits_{q} & \max\limits_{i=1,\ldots,k} \parallel q_i - p_i \parallel_2 \\ \text{such that} & Aq = 0 \end{array} \tag{12}$$

for our *minimax* distance metric defined in Problem 3.2.1, and for our total distance metric, we have

$$\begin{array}{ll} \min\limits_{q} & \sum\limits_{i=1}^{k} \parallel q_i - p_i \parallel_2 \\ \text{such that} & Aq = 0 \end{array} \tag{13}$$

where $Aq = 0$ denotes the set of constraints defined in (10)-(11).

While both the constraints and objective functions of these problems are convex, the form of the latter does not lend itself to traditional optimization techniques. To remedy this, we augment the state vector q with an auxiliary variable t_1 for our minimax metric, and with k auxiliary variables $t = (t_1, \ldots, t_k)^T$ for our total distance metric. The optimization problems in (12)-(13) can then be restated as

$$\begin{array}{ll} \min\limits_{q,t_1} & t_1 \\ \text{such that} & \parallel q_i - p_i \parallel_2 \leq t_1, \; i = 1, \ldots, k \\ & Aq = 0 \end{array} \tag{14}$$

and for our total distance metric

$$\begin{array}{ll} \min\limits_{q,t} & \sum\limits_{i=1}^{k} t_i \\ \text{such that} & \parallel q_i - p_i \parallel_2 \leq t_i, \; i = 1, \ldots, k \\ & Aq = 0 \end{array} \tag{15}$$

Both forms are equivalent. However, the objective functions in (14)-(15) are now twice differentiable. We have also introduced k additional second-order cone constraints which are still convex. The problem is now in the form of a second-order cone program (SOCP), which has a unique, globally optimal solution. It can be solved very efficiently using modern interior point algorithms [13].

Simulation Results: Fig. 1 shows a simulation trial demonstrating the process. In this example, 55 nodes were tasked with transitioning to a new shape while minimizing the maximum distance that any node must travel. While deliberately contrived, this example demonstrates the efficacy of our approach. The formation is able to optimally transition from an arbitrary shape to a very specific shape. This would typically be the case when a formation of robots was initially deployed. In this example, none of the shape parameters (*i.e.*, translation, rotation or scale) was regulated.

V. Optimal Solutions in $\mathbb{R}^2/\mathbb{R}^3$

We now consider the specific case where the orientation of the shape is fixed. That is

$$[S]_{PS} = \{\alpha S + 1_k d^T : \alpha \in \mathbb{R}_+, d \in \mathbb{R}^m\} \tag{16}$$

which is defined as the *pre-shape* of S. The constraints now implied by this new equivalence class are of the form

$$q_i - q_1 = \alpha s_i, \quad i = 2, \ldots, k \tag{17}$$

which are linear in terms of our objective position and scale.

This allows us to freely regulate the formation scale. We assume then that $\alpha_{min} \leq \alpha \leq \alpha_{max}$, $\alpha_{min} \in \mathbb{R}_+$. For the case of a fixed scale, we need only set $\alpha_{min} = \alpha_{max}$. The problem of finding the formation pose Q can now be posed

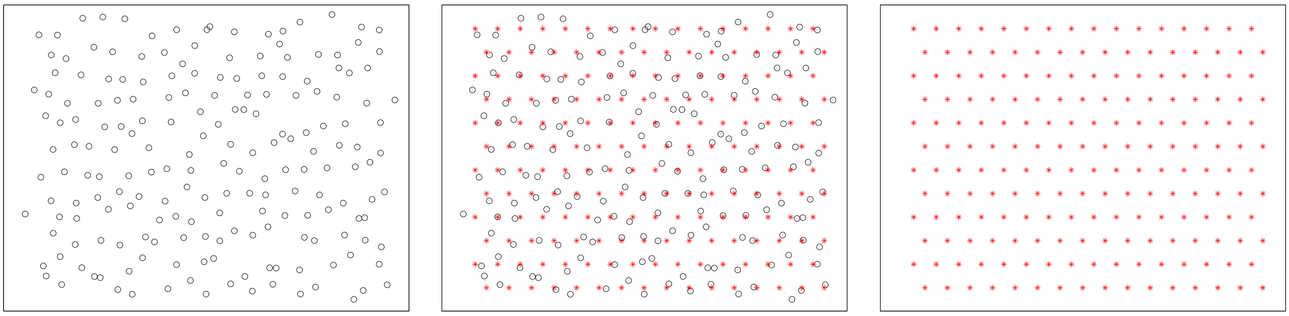

Fig. 2. The initial dispersion of a set of 192 nodes in a MANET (left). The final formation trajectories and shape that achieve the desired $\{3,6\}$ tessellation while minimizing the combined distance that all nodes must travel (center, right). In this example, both the desired orientation ($\theta = 0$) and scale ($\alpha = 1$) are fixed.

as the constrained optimization problems

$$\begin{array}{lll} \min\limits_{q,\alpha,t_1} & t_1 & \\ \text{such that} & \| q_i - p_i \|_2 \leq t_1, & i = 1,\dots,k \\ & q_i - q_1 = \alpha s_i, & i = 2,\dots,k \\ & -\alpha \leq -\alpha_{min} & \\ & \alpha \leq \alpha_{max} & \end{array} \tag{18}$$

and for our total distance metric

$$\begin{array}{lll} \min\limits_{q,\alpha,t} & \sum\limits_{i=1}^{k} t_i & \\ \text{such that} & \| q_i - p_i \|_2 \leq t_i, & i = 1,\dots,k \\ & q_i - q_1 = \alpha s_i, & i = 2,\dots,k \\ & -\alpha \leq -\alpha_{min} & \\ & \alpha \leq \alpha_{max} & \end{array} \tag{19}$$

This problem is again a SOCP. We should emphasize that unlike in Section IV, no assumptions were made in this formulation with regards to the degree of m. As such, it provides optimal solutions in both $\mathbb{R}^2$ and $\mathbb{R}^3$. Also note that although we are optimizing over the state vector $x = (q_1, \dots, q_k, \alpha)^T$, the $m(k-1)$ constraints $q_i - q_1 = \alpha s_i, i = 2, \dots, k$ are sufficient to ensure the rigidity of the formation. As such, we are in fact optimizing over only q_1 and α, which correspond to the translation and scale terms of (16).

Simulation Results: Fig. 2 shows a simulation trial demonstrating the efficacy of the approach. In this example, a mobile ad-hoc network (MANET) of 192 nodes is initially dispersed on the plane. The desired shape is a $\{3,6\}$ tessellation where both the scale and orientation are fixed. The objective is to minimize the combined distance that the nodes must travel to reach the desired shape. The motivation for this would be to allow some portion of the network to remain active for as long as possible. The resulting trajectories and final formation shape are also shown.

VI. On Regulating Scale

From our definition of shape, the scale α is constrained to the interval $(0, +\infty]$. This can potentially result in a formation size that is too large - possibly violating sensing or communication constraints. Alternately, for objective shapes that are dramatically different from the current formation configuration (*e.g.*, a reflection) the optimal shape may be for $\alpha \to 0$. As a consequence, in practice it may be appropriate to define a scale range $\alpha_{min} \leq \alpha \leq \alpha_{max}$, $\alpha_{min} \in \mathbb{R}_+$, or even to fix the scale so that a more constrained formation shape can be achieved.

For fixed orientation, the constraints are linear in α (Equation 17), and as a consequence regulating scale is accomplished by bounding α as discussed in Section V. However, for the more general case discussed in Section IV the problem is far less trivial.

Consider regulating the scale for a shape over $SE(2)$. Establishing an upper bound α_{max} for the scale is straightforward. From our definition above we have

$$\| q_2 - q_1 \| \leq \alpha_{max} \| s_2 \| \tag{20}$$

This is also a second-order cone constraint. However, adding a minimum bound or equality constraint on α renders the problem significantly more difficult as the constraints

$$\begin{array}{rcl} \| q_2 - q_1 \| & \geq & \alpha_{min} \| s_2 \| \\ \| q_2 - q_1 \| & = & \alpha \| s_2 \| \end{array} \tag{21}$$

are *not* convex. For these cases, randomization and linearization/convex restriction techniques can be applied to find approximate solutions [14]. However, randomization schemes are inappropriate in the presence of equality constraints. Additionally, the complete lack of convexity in our lower bound constraint will leave relaxation solutions highly dependent upon the shape orientation of the initial feasible point Q_0 provided. Instead, we propose our own approximation scheme.

Since orientation in $SE(2)$ is constrained to the interval $[0, 2\pi)$, the ability to obtain an optimal solution for fixed orientations in $\mathbb{R}^2$ also provides a mechanism for approximating optimal solutions in $SE(2)$. We can accomplish this by constructing a family of shape icons $\mathbf{S} = \{S_1, \dots, S_z\}$ corresponding to a discretization of orientations $\theta = \{\theta_1, \dots, \theta_z\}$ such that

$$S_i = S_1 R(\theta_i - \theta_1), \; i = 1, \dots, z \tag{22}$$

where $R(\theta_i - \theta_1)$ denotes the rotation matrix relating the shape icon S_i to S_1.

By modeling each θ_i as a constant, we can approximate the optimal solution by solving these z SOCP in accordance with the procedure in Section V above. The resulting shape $Q_j,\ j \in 1,\ldots,z$ corresponding to the shape icon S_j with minimum objective will be our solution. The exactness of such an approach will only depend upon the resolution for the discretization of θ, as the solution for each θ_i will be optimal.

Simulation Results: A series of simulation trials was conducted in an attempt to empirically characterize the performance of this approximation approach. Three different objective shapes $S_i,\ \ i \in 1,\ldots,3$ were chosen corresponding to the three regular tessellations on the plane $\{3,6\},\{4,4\}$ and $\{6,3\}$, respectively [15]. For each trial j, the initial shape P_j was obtained by perturbing the position of each node in the objective shape S_i with a displacement randomly sampled from a two-dimensional Gaussian distribution.

For a performance baseline, the optimal formation pose was first obtained in accordance with Section IV. Our approximation scheme then attacked the same problem by solving the set of SOCPs for the family of shape icons obtained from discretizing θ in accordance with (22). To be consistent, α was unconstrained in both cases.

A total of 100 trials with 12 nodes was conducted for each S_i. A statistical summary of the results for orientation discretizations of $1°, 5°, 10°$ and $20°$ is provided at Table I. Note that the values are normalized against the optimal distances obtained from our baseline simulations. Thus, an entry of 1.1 denotes that the *minimax* distance achieved from the approximation scheme was 10% greater than the optimal distance.

TABLE I

NORMALIZED DISTANCES FOR *minimax* APPROXIMATION

Objective Shape	$d\theta$ (degrees)	Mean Distance	Median Distance	Max. Distance
$\{3,6\}$	1	1.001	1.002	1.005
$\{3,6\}$	5	1.007	1.003	1.040
$\{3,6\}$	10	1.016	1.012	1.076
$\{3,6\}$	20	1.046	1.030	1.318
$\{4,4\}$	1	1.001	1.000	1.006
$\{4,4\}$	5	1.001	1.004	1.040
$\{4,4\}$	10	1.015	1.010	1.066
$\{4,4\}$	20	1.044	1.020	1.281
$\{6,3\}$	1	1.001	1.000	1.005
$\{6,3\}$	5	1.009	1.005	1.044
$\{6,3\}$	10	1.023	1.012	1.136
$\{6,3\}$	20	1.053	1.032	1.298

As expected, performance is strongly tied to the resolution of θ. For an orientation discretization of $1°$, results of all trials were within 1% of the optimal solution in each category. While mean and median performance for higher resolutions remains quite good for discretizations up to $20°$, the errors can still be significant at discretizations as low $5°$. While performing a large number of iterations is far from ideal, this approach may prove useful for the case where lower bounds on the formation scale must be explicitly constrained.

VII. ON INTEGRATING MOTION CONSTRAINTS

This framework relies upon the convexity of the underlying problem to generate optimal shape transitions for large formations (*e.g.* > 1000 nodes) very quickly (*e.g.* ≈ 1 second) using current PC technology. As a consequence, motion constraints can also be accommodated so long as they can be expressed similarly. As an example, if the motion of agent i was constrained to a maximum distance d_{max}, this can also be expressed as a second-order cone constraint of the form

$$\| q_i - p_i \| \leq d_{max} \tag{23}$$

This would allow less mobile (or fixed) nodes to be accommodated. Others might be expressed in terms of linear constraints. As an example, a formation wanting to maintain positive velocity in the x-direction could ensure this by specifying a minimum forward distance traveled d_{min} for each node as

$$q_i^x - p_i^x \geq d_{min},\ i \in 1,\ldots,k \tag{24}$$

In summary, if the motion constraints can be expressed in terms of some combination of *feasible* linear, second-order cone, or semidefinite constraints, they can be directly integrated within our framework.

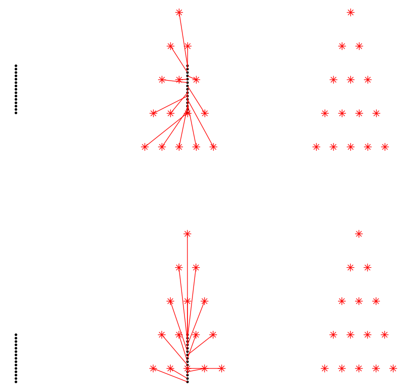

Fig. 3. Shape transitions for a team of 15 robots with no motion constraints (top row) and with $q_i^y - p_i^y \geq 0, i \in 1,\ldots,k$ (bottom row). In both instances, the objective was to migrate from the initial inline shape (black circles) to a triangular shaped formation (red stars). As expected, the additional constraints for the latter increased the minimax distance for the formation (by 87% in this case).

Simulation Results: A sample simulation trial showing formation shape changes without (top row) and with (bottom row) motion constraints is shown at Fig. 3. In both cases, the objective was for the team of 15 robots to transition from an initial inline shape to a triangular shaped formation. Optimization was over our minimax metric. For the latter case, the minimum forward distance traveled for each node was constrained as $q_i^y - p_i^y \geq 0,\ \ i \in 1,\ldots,k$. Scale and orientation were fixed in both cases. As expected, the objective shape was reached in both instances. However, the additional constraints for the latter case increased the minimax distance for the formation (by 87% in this example).

VIII. On Complexity

The complexity of the framework corresponds to that of the underlying optimization problem in which it was posed. The SOCP formulation requires $O(\sqrt{k})$ iterations to reduce the duality gap to some constant fraction of itself. For general problems, the amount of work for each iteration is $O(k^3)$, for a total complexity of $O(k^{3.5})$ [13]. However, our problem is very structured. The format of the constraints representing both shape and distance is invariant to formation size. Furthermore, the constraints are inherently sparse, with each involving only $O(m)$ variables regardless of the size of k. Exploiting both of these characteristics can result in a significantly reduced per-iteration complexity, with $O(k^2)$ being a more likely bound. Empirical results in [16] imply that problems of > 1000 nodes can be solved in ≈ 1 second with current PC technology and efficient software implementations.

Furthermore, these algorithms lend themselves to distributed computation. Results from parallel implementations indicate a near linear speedup is possible [17]. This of course comes at the expense of communication requirements, but would permit our approach to be implemented on embedded class processors for reasonably large formations.

IX. Discussion and Conclusions

In this paper, we have devised a framework for repositioning a formation of robots to a new shape while minimizing either the maximum distance that any robot travels, or the total distance traveled by the formation. Optimal solutions in $SE(2)$ and $\mathbb{R}^3$ for fixed orientation were achieved for both metrics through second-order cone programming (SOCP) techniques. Recent advances in interior point methods for solving such problems can allow optimal positions for formations of > 1000 robots to be solved very efficiently (in ≈ 1 second). We feel that these results will be useful in formation control and mobile adhoc network deployments.

Despite these results, there are significant opportunities for future work. While we achieve optimal solutions in $SE(2)$, we are only able to regulate an upper bound on the formation scale α. In many applications, a lower bound or fixed value for α may be desirable while still optimizing over shape orientation. We are investigating alternate definitions of scale in an attempt to achieve such a capability. We shall also evaluate convex relaxation techniques and contrast these results to our current approximation algorithm.

An optimal solution in $SE(3)$ would also appear to be a next logical step. However, the ability to extend our results from $SE(2)$ is not immediately obvious. We will take a more formal look at this problem in the near future.

We shall also look at extending our definition of shape to include assignment. The optimal solution to [10] can be achieved through a linear programming formulation. Thus, both problems can be posed as convex optimization problems. We are currently looking at the potential for merging these into a single convex formulation.

While our results generate optimal positions, the subject of robot control has been heretofore ignored. We are currently investigating extending these results by regulating velocity profiles via motion constraints in order to ensure collision free trajectories. We will also investigate integrating our current results into more traditional control approaches.

Finally as an extension, the emergent field of nanoassembly (*i.e.*, the programming and coordination of large number of nanorobots [18]) may benefit from the results presented herein.

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